Suppression of intensity noise of a laser-diode-pumped single-frequency Nd:YVO₄ laser by optoelectronic control

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The intensity-noise reduction of a laser-diode-pumped single-frequency ring Nd:YVO₄ laser when different optoelectronic control systems are used is theoretically and experimentally investigated. It has been demonstrated that combining two techniques, optoelectronic feedback control of the drive current of the pump laser diode and feed-forward control of the output laser beam, is a good way to significantly suppress the intensity noise of a laser at low frequency. © 2003 Optical Society of America

1. Introduction

Single-frequency laser sources with low intensity noise are useful for many applications such as high-sensitivity measurements, high-precision interferometry, precision spectroscopy, and optical communications. Laser-diode-pumped (LD-) pumped solid-state lasers and intracavity frequency doublers are well known as efficient sources of intensity-stable single-frequency radiation.¹⁻⁴ In practical LD-pumped single-frequency laser systems, however, the intensity noise spectrum has a resonance, that is an underdamped driven second-order oscillation, that is known as resonant relaxation oscillation (RRO).⁵ The low-frequency part of the intensity-noise spectrum below the RRO is determined by the laser’s pump noise, whereas the RRO is driven by vacuum fluctuations, dipole fluctuations, and intracavity losses. Significant suppression of relaxation oscillations in diode-pumped single-frequency lasers has been achieved in various ways, such as stabilizing laser intensity by means of electronic feedback loops,⁶⁻⁹ injection locking the laser to an intensity-stable master laser,¹⁰,¹¹ and combining these techniques.¹² The intensity noise at low frequencies can be reduced by suppression of the pump noise.¹³,¹⁴ Unlike in the single-frequency laser, in the intensity noise spectrum of the single-frequency-doubling laser the RRO is not present, but there is still a large noise that results from an overdamped driven second-order oscillator during nonlinear conversion.¹⁵ And the intensity noise of the single-frequency intracavity frequency-doubling laser was reduced with an optoelectronic feedback circuit inserted directly into the pump current.¹⁶,¹⁷

The pump noise transfer function of a LD-pumped single-frequency laser is an underdamped driven second-order oscillator. Significant suppression of RRO by an optoelectronic feedback circuit inserted directly into the pump current is well known.⁶⁻⁹ However, there are a few technical problems in this system that limit further improvement of its intensity stability. The primary problem associated with this system is the sharp 180° phase change across the RRO, which, in practice, is electronically compensated with difficulty; thus it is technically challenging to reach the high-gain limit for a given system. An alternative method for suppressing the intensity noise of a laser is to insert an amplitude modulator (AM) controlled by the optoelectronic feedback into the laser beam.¹⁸,¹⁹ This feedback loop is designed easily and can reach high gain because no underdamped driven second-order oscillator is included in it. But the RRO peak cannot be suppressed effectively by this method. Optoelectronic feed-forward acting on the AM has been used for producing perfect noiseless signal amplification.²⁰ In this paper we describe a highly effective system for suppressing the intensity noise of a LD-pumped single-frequency Nd:YVO₄ laser. Based on analyzing the differences between the intensity-noise spectra suppressed by means of feedback and feed-forward, we exploit two techniques, an optoelectronic feedback circuit in-
asserted directly into the drive current of the laser diode and a feed-forward loop acting on an AM inserted into the laser beam in the system. Because of the combination of the two techniques, both the RRO and the intensity noise in low frequencies have been reduced significantly.

2. Feedback and Feed-forward Control

Schematics of feedback and feed-forward loops are shown in Figs. 1(a) and 1(b), respectively. A small amount of light split from the laser beam by a polarizing beam splitter is detected by a photoelectric detector. Then the signal is amplified by an electronic feedback loop. The amplified electronic signal is imposed on the AM placed in the laser beam. We can write the input laser beam in the linearized form

$$\hat{A}_{in}(t) = A_{in} + \delta \hat{A}_{in}(t),$$

where \(\hat{A}_{in}\) is the field annihilation operator, \(A_{in}\) is the classic steady-state value of the field, and \(\delta \hat{A}_{in}\) is a zero-mean operator that includes all the classical and quantum noises. After passing the AM in the feedback loop, the laser field is written as

$$\hat{A}_{in}^\prime(t) = \hat{A}_{in}(t) + \delta \hat{f},$$

where \(\delta \hat{f}\) is a fluctuating term added in the laser field by the AM, which does not affect the steady-state value of the field. The laser beam is then split by a beam splitter with intensity transmittance \(\varepsilon\). The reflected beam is incident upon the in-loop photodetector with efficiency \(\eta_{1}\) to produce the feedback current. The transmitted beam is the output field (out of loop). The out-of-loop and the in-loop fields are given by

$$\hat{A}_{outLP}(t) = \sqrt{\varepsilon} \hat{A}_{in}^\prime(t) - \sqrt{1 - \varepsilon} \delta \hat{v}_{1},$$

$$\hat{A}_{inLP}(t) = \sqrt{\eta_{1}}(\sqrt{1 - \varepsilon} \hat{A}_{in}^\prime(t) + \sqrt{\varepsilon} \delta \hat{v}_{1})$$

$$+ \sqrt{1 - \eta_{1}} \delta \hat{v}_{2},$$

where vacuum fluctuations \(\delta \hat{v}_{1}\) and \(\delta \hat{v}_{2}\) stand for the noises from the beam splitter and the in-loop photodetector, respectively. According to the law of energy conservation, vacuum fluctuations \(\delta \hat{v}_{1}\) introduced into the reflected beam should be anticorrelated with those into the transmitted beam. Photocurrent \(i_{inLP}\) may be expressed in terms of the detected in-loop field as

$$i_{inLP} = \hat{A}_{in}^\prime(t) \hat{A}_{inLP}(t).$$

When we retain only first-order fluctuation terms, the fluctuations of the in-loop photocurrent are given by

$$\delta i_{inLP} = A_{in}^\prime \sqrt{\eta_{1}}(1 - \varepsilon_{1})(\sqrt{\eta_{1}}(1 - \varepsilon_{1}) \delta \hat{X}_{\hat{A}_{in}}$$

$$+ \sqrt{\eta_{1}} \varepsilon_{1} \delta \hat{X}_{\hat{A}_{t}} + \sqrt{1 - \eta_{1}} \delta \hat{X}_{\hat{V}_{in}}$$

$$+ \sqrt{1 - \eta_{1}} \delta \hat{V}_{in},$$

where \(\delta \hat{X}_{\hat{A}_{in}}\), \(\delta \hat{X}_{\hat{A}_{t}}\), and \(\delta \hat{X}_{\hat{V}_{in}}\) are the amplitude fluctuations that correspond to the input field, to the vacuum noise from the beam splitter \(\delta \hat{V}_{1}\), and to the nonunity detector efficiency \(\delta \hat{V}_{2}\), where \(\delta \hat{X}_{\hat{A}_{in}} = \delta \hat{A}_{in}^\prime(t) + \delta \hat{A}_{in}^\prime(t)\) and \(\delta \hat{X}_{\hat{V}_{in}} = \delta \hat{V}_{1} + \delta \hat{V}_{2}\). Fluctuation field \(\delta \hat{f}\) may be expressed as a convolution of the time response of the feedback electronics, \(k(t)\), and the ac component of the in-loop photocurrent field, \(\delta i_{inLP}(t - \tau)\):

$$\delta \hat{f} = - \int_{-\infty}^{\infty} k(\tau) \delta i_{inLP}(t - \tau) d\tau$$

$$= - \int_{-\infty}^{\infty} k(\tau) A_{in}^\prime \sqrt{\eta_{1}}(1 - \varepsilon_{1})[\sqrt{\eta_{1}}(1 - \varepsilon_{1}) \delta \hat{X}_{\hat{A}_{in}}$$

$$\times (t - \tau) + \sqrt{\eta_{1}} \varepsilon_{1} \delta \hat{X}_{\hat{A}_{t}}(t - \tau)$$

$$+ \sqrt{1 - \eta_{1}} \delta \hat{X}_{\hat{V}_{in}}(t - \tau)] d\tau,$$

where the minus is involved with negative feedback. The amplitude fluctuation spectrum of the output field \(\hat{A}_{outLP}(t)\) is the expectation value of the Fourier transform of the absolute squared amplitude fluctuations, i.e., \(V_{outLP}(\omega) = \langle |\delta \hat{X}(\omega)|^{2} \rangle\). In experiments, we obtained \(V_{outLP}\) by normalizing the power spectrum with a spectrum analyzer to the quantum-noise limit (QNL), which is the noise spectrum of a coherent state with the same optical power. Equations (2), (3), and (6) may be solved in Fourier space to give
the amplitude fluctuation spectrum of the output field $A_{\text{outLP}}(t)$:

$$V_{\text{feedback}}(\omega) = \frac{\varepsilon_1}{1 + h(\omega)^2} V_{\text{in}}(\omega) + \frac{|\varepsilon_1 - h(\omega) - 1|^2}{(1 - \varepsilon_1)|1 + h(\omega)|^2} V_{\text{bs1}} + \frac{\varepsilon_1(1 - \eta_1)|h(\omega)|^2}{(1 - \varepsilon_1)|1 + h(\omega)|^2} V_{\text{bs2}}$$

$$= 1 + \frac{(1 - \varepsilon_1)\varepsilon_1\eta_1[V_{\text{in}}(\omega) - 1] + \varepsilon_1|h(\omega)|^2}{(1 - \varepsilon_1)^2|1 + h(\omega)|^2},$$

where the transfer function $h(\omega) = k(\omega)A_{\text{opt}}\eta_1(1 - \varepsilon_1)$ of the feedback system summarizes the effects from the beam splitter, the control electronics, and the AM and the noises from the beam splitter and the non-unity detector efficiency are on the level of the QNL: $V_{\text{bs1}} = V_{\text{bs2}} = 1$.

In the feed-forward loop a beam splitter of transmissivity $\varepsilon_2$ is placed in the laser beam before the AM [Fig. 1(b)]. Similarly, the out-of-loop and the detected in-loop fields are given by

$$\hat{A}_{\text{outLP}}(t) = \sqrt{\varepsilon_2}\hat{A}_{\text{in}}(t) - \sqrt{1 - \varepsilon_2}\delta v_1,$$

$$\hat{A}_{\text{inLP}}(t) = \sqrt{\eta_1}\sqrt{1 - \varepsilon_2}\hat{A}_{\text{in}}(t) + \sqrt{\varepsilon_2}\delta v_1 + \sqrt{1 - \eta_2}\delta v_2.$$

After it has passed through the AM, the out-of-loop field is written as

$$\hat{A}_{\text{outLP}}'(t) = \hat{A}_{\text{outLP}}(t) + \delta \hat{r},$$

where $\delta \hat{r}$ is a small fluctuation term added in the laser field. Solving Eqs. (9), (8), and (6) in Fourier space, we obtain the amplitude fluctuation spectrum of the output field $\hat{A}_{\text{outLP}}'(t)$:

$$V_{\text{outLP}}(\omega) = |\sqrt{\varepsilon_2} - h(\omega)|^2V_{\text{in}}(\omega) + |\sqrt{1 - \varepsilon_2} + h(\omega)|^2V_{\text{bs1}}$$

$$+ |h(\omega)|^2V_{\text{bs2}}.$$

Minimizing $V_{\text{feedback}}(\omega)$ and $V_{\text{outLP}}(\omega)$, we get the optimum gain of the feedback and the feed-forward loops for suppressing classic noise:

$$h_{\text{opt feedback}}(\omega) = \eta_1(1 - \varepsilon_1)(V_{\text{in}} - 1)$$

$$h_{\text{opt forward}}(\omega) = \eta_1(1 - \varepsilon_1)(\hat{\varepsilon}_2(V_{\text{in}} - 1) + \eta_1(1 - \varepsilon_1)(V_{\text{in}} - 1)).$$

With the optimum gains, the minimum out-of-loop noise spectrum is obtained:

$$V_{\text{feedback}}^{\text{opt}}(\omega) = V_{\text{forward}}^{\text{opt}}(\omega)$$

$$= 1 + \frac{\varepsilon_1(V_{\text{in}} - 1)}{1 + \eta_1(1 - \varepsilon_1)(V_{\text{in}} - 1)}.$$

The feedback and the feed-forward loops have the same minimum out-of-loop noise spectrum, which is above the QNL. This is a fundamental limit to the performance of an intensity-stabilization loop. When the classic noise $V_{\text{in}} - 1$ less than $1/(1 - \varepsilon_1)$, the noise is not suppressed by the control loop. The noises relative to the QNL of the out-of-loop light as a function of the loop gains at different laser noises for the feedback and the feed-forward loops are shown in Fig. 2. For the feedback loop, large laser noise needs large gain for maximum suppression [see Eq. (11)]. Further attempts to suppress the remain-
ing laser noise by increasing the gain will actually cause the noise to rise again. In the high-gain limit the out-of-loop noise is found to be

$$
\lim_{H \to \infty} (V_{\text{feedback}}) = 1 + \frac{\varepsilon_1}{\eta_1(1 - \varepsilon_1)},
$$

which depends only on beam splitter ratio $\varepsilon_1$ and in-loop detector efficiency $\eta_1$. For a Poissonian vacuum at the beam splitter ($V_{\text{in}} = 1$) the high-gain limit of the out-of-loop noise is always greater than 1; i.e., it is always super-Poissonian. This result is in agreement with those calculated from other quantum-feedback models.\(^9\) The cause of this behavior lies in the vacuum fluctuations introduced by the beam splitter. The noise that is due to the beam splitter in the out-of-loop field is out of phase with the noise introduced in the in-loop field. Negative feedback therefore amplifies the beam-splitter vacuum noise in the out-of-loop field, preventing sub-Poissonian intensity statistics. For the feed-forward loop the gain for maximum suppression reaches a limit with increasing laser noise:

$$
\lim_{V_{\text{in}} \to \infty} [H_{\text{feedforward}}(\omega)] = \sqrt{\varepsilon_1}. \quad (15)
$$

For the gain $H_{\text{feedforward}}(\omega) = \sqrt{\varepsilon_1}$, the laser input noise term (the first term) in Eq. (10) is eliminated and only the vacuum input noise term is retained; thus the out-of-loop noise spectrum of the feed-forward loop is equal to that of feedback loop in the high-gain limit:

$$
\lim_{H(\omega) \to \sqrt{\varepsilon_1/\eta_1(1 - \varepsilon_1)}} (V_{\text{forward}}) = \lim_{H(\omega) \to \infty} (V_{\text{feedback}}) = 1 + \frac{\varepsilon_1}{\eta_1(1 - \varepsilon_1)}. \quad (16)
$$

When we further increase the gain, the out-of-loop noise spectrum will rise quickly. Figure 3 shows the optimum gains for (i) the feedback and (ii) the feed-forward loops as a function of laser input noise $V_{\text{in}}$. $\varepsilon_1 = \eta_1 = 0.95$.

The feedback loop. Usually, in practical systems laser noise is significant and the in-loop electronic gain can not be made high; thus use of a feed-forward loop should be preferable. However, time delays are actually included in the feedback and feed-forward loops, which have an important influence on suppressing noise, as they introduce a phase lag. Figure 4 shows the effect of the phase lag in the loop on noise suppression. Noise suppression with the feed-forward loop is more sensitive to phase lag than that with the feedback loop.

3. Experiment

Figure 5 is a schematic of the diode-pumped single-frequency Nd:YVO$_4$ laser used in our experiments.
The unidirectional ring laser is pumped by a laser diode through an optical coupling system. Input mirror M1 has antireflection coating at 808 nm on both internal and external facets and high-reflection coating at 1064 nm on the internal facet. Concave mirrors M4 and M3 have high reflectivity at 1064 nm. A terbium gallium garnet (TGG) crystal and a half-wave plate (λ/2) are placed in the cavity as an optical diode to enforce unidirectional operation. Output coupler M2 has 96% reflectance at 1064 nm. The pump power that corresponds to the lasing threshold is 200 mW. The output power of the laser is 350 mW at 1064 nm with a pump power of 1.5 W.

A schematic of the experimental arrangement for noise control and monitoring of a laser is shown in Fig. 6. In-loop and out-of-loop detectors D1 and D2 use Epitax 300 InGaAs photodiodes. The beam splitter in Fig. 6 consists of a rotatable half-wave plate and a polarizing beam splitter that may adjust the beam splitter ratio ε1. D1 (Analog Modules 714A), which also includes an electronic amplifier, has a large gain and broad bandwidth from 10 kHz to ~100 MHz. The power of ~1 mW is detected by D1; then the photocurrent amplified by a Mini-Circuits ZHL-6A amplifier acts on an electro-optic modulator (New Focus 4104), which, along with a polarizing beam splitter, forms an AM. The gain of the in-loop is controlled by variable attenuators (Trilithic RA-50-BNC). A transimpedance optical amplifier circuit in D2 is used to convert the photocurrent to voltage. Detector D2 samples a small fraction of the out-of-loop laser from a beam splitter, and the detected photocurrent is ~1.3 mA. The power spectrum of the detected signal is recorded by a Hewlett-Packard HP-8890L spectrum analyzer. The spectra of the QNL are given by means of white-light illumination; each produces the same amount of photocurrent (1.3 mA). \(^{21}\)

Figure 7 shows the noise power spectra of the out-of-loop fields when the in-loop gain is adjusted to achieve the maximum intensity-noise suppression for the feedback and feed-forward loops. Curves (i), (ii), and (iii) are the spectra for free running, i.e., \(h(\omega) = 0\), feedback, and feed-forward, respectively. Noise suppression with feedback is better than that with feedback at the low frequency because when the electronic circuit has been saturated the optimum gain required by the feedback loop is still not reached. This means that the optimum gain required by the feedback loop cannot be provided by our experimental setup; however, the smaller gain required by the feed-forward loop may be satisfied easily. Figure 8 shows the noise spectra (i) free running, and with the feed-forward loop with (iii) the optimum gain of 3 dB, (ii) less than the optimum gain, and (iv) more than the optimum gain. For gain that is less or larger than the optimum gain, the noise suppression becomes worse. However, the change in the noise spectrum is small near the optimum gain when the feedback loop is used. The experimental results agree with the theoretical expectation from Fig. 2. The peak of RRO cannot be reduced effectively by the optoelectronic forward or feed-forward loop acting on the AM.

To reduce the peak of RRO we combine two techniques for optoelectronic control of the AM in the laser beam and the pump current of the LD as shown in Fig. 6. First, significant suppression of RRO is achieved by an optoelectronic circuit inserted directly

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**Figure 6.** Experimental arrangement for combining the two techniques of the optoelectronic noise control. FW’s, feed-forward loop; D1, D2, detectors.

**Figure 7.** Feedback and feed-forward experimental noise spectra: (i) free running, (ii) feedback, (iii) feed-forward, (iv) the QNL.

**Figure 8.** Noise spectra with feed-forward loops of several gains: (i) free-running, (ii) less than the optimum gain of 3 dB, (iii) the optimum gain, (iv) larger than the optimum gain of 3 dB, (v) the QNL.
The feed-forward loop needs smaller gain to obtain maximum noise reduction for large noise than the feedback loop needs; however, the maximum noise suppression of the feed-forward loop is more sensitive to the phase lag than that of the feedback loop. We compared the noise suppression spectra of a LD-pumped single-frequency Nd:YVO$_4$ laser by means of feedback and feed-forward loops. The feed-forward loop can effectively reduce intensity noise at the low frequency; however, the peak RRO cannot be suppressed significantly. We further suppressed the RRO of a laser by combining an optoelectronic feedback circuit inserted directly into the drive current of the LD and the feed-forward loop whose control element is an AM in a laser beam. It has been shown that the RRO and the intensity noise in low frequencies were reduced significantly, by $\sim 40$ and $\sim 20$ dB, respectively.

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